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SHORTER COMMUNICATION

RADIANT HEAT TRANSFER TO A DILUTE CLOUD OF PARTICLES

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NOMENCLATURE

- A_c, outside area of cloud;
- D_h, hydraulic diameter of cloud $(4V_c/A_c)$;
- D_p, particle diameter;
- K, extinction coefficient, or projected particle area per unit volume $(n\pi D_p^2/4)$;
- L, mean beam length as defined by equation (9);
- L', mean beam length from particle to cloud boundary;
- n, particle density;
- q, radiant heat transfer to each particle;
- Q, radiant heat transfer to whole cloud;
- r, ratio between successive absorptions;
- S, view factor between particle and surroundings;
- T_p, temperature of particles;
- T_∞, temperature of surroundings;
- V_c, volume of cloud;
- x, distance between two points on cloud boundary.

Greek symbols

- α_c, absorptivity of cloud;
- α_p, absorptivity of particle;
- σ, Stefan–Boltzmann constant.

INTRODUCTION

THE PROBLEM of radiant heat transfer to clouds of droplets or particles is important in such equipment as combustion chambers and spray dryers. When the perimeter of the particles is large compared with the wavelength, the cloud absorptivity, α_c, is given by [1]

$$\alpha_c = 1-\exp(-KL) \tag{1}$$

where K is the projected area of the particles in unit volume of space, and L is the mean beam length. When the particles are grey, only a fraction α_p of the intercepted light is absorbed, and the formula normally used to date is [1]

$$\alpha_c = 1-\exp(-\alpha_p KL). \tag{2}$$

Equation (2) is inaccurate as it assumes that non-absorbed radiation continues to travel in the same direction after impinging on the particle. In the following, an attempt will be made to account more accurately for the effect of multiple reflections in a dilute cloud of isotropically reflecting particles.

FORMULA FOR CLOUD ABSORPTIVITY

Consider a beam of light of unit intensity falling on a cloud. It can readily be shown that a fraction $\{1-\exp(-KL)\}$ of the light is intercepted by the particles. Of this intercepted light, a fraction α_p is absorbed and $(1-\alpha_p)$ is reflected.

$$\text{First absorption} = \{1-\exp(-KL)\}\alpha_p \tag{3}$$

$$\text{First reflection} = \{1-\exp(-KL)\}(1-\alpha_p). \tag{4}$$

The light from the first reflection travels another mean beam length L' ($L' \neq L$) before reaching the cloud boundary. Along similar lines of reasoning, it can be seen that

$$\begin{aligned}&\text{Second-time intercepted radiation}\\&\quad= \text{First reflection} \times \{1-\exp(-KL')\}\\&\quad= \{1-\exp(-KL)\}(1-\alpha_p)\{1-\exp(-KL')\}\\&\text{Second absorption}\\&\quad= \text{Second-time intercepted radiation} \times \alpha_p\\&\quad= \{1-\exp(-KL)\}(1-\alpha_p)\{1-\exp(-KL')\}\alpha_p.\end{aligned} \tag{5}$$

Assuming that L' remains constant for the third, fourth ... reflections and interceptions, it can be seen that the ratio r between successive absorptions is, from equations (3) and (5):

$$r = (1-\alpha_p)\{1-\exp(-KL')\}. \tag{6}$$

To find the total absorption and hence the cloud absorptivity α_c, we sum up the successive absorptions:

$$\begin{aligned}\alpha_c &= \text{total absorption}\\&= \text{first absorption} \times \sum_{j=0}^{\infty} r^j\\&= \frac{\alpha_p\{1-\exp(-KL)\}}{1-(1-\alpha_p)\{1-\exp(-KL')\}}.\end{aligned} \tag{7}$$

Assuming that the mean beam length from boundary to boundary L is twice the mean beam length from particle to boundary L' [as will be seen later, this assumption is required for the consistency of equation (7) at low KL], we finally obtain:

$$\alpha_c = \frac{\alpha_p\{1-\exp(-KL)\}}{1-(1-\alpha_p)\{1-\exp(-KL/2)\}}. \tag{8}$$

CALCULATION OF THE MEAN BEAM LENGTH

The mean beam length L is defined by [2, 3]

$$\exp(-KL) = \frac{\int_{2\pi}\exp(-Kx)\times(\text{view angle factor})\times d(\text{solid angle})}{\int_{2\pi}(\text{view angle factor})\times d(\text{solid angle})} \tag{9}$$

Hottel [2] showed that for this equation to hold within a few per cent, it is sufficient to take

$$L = 0.9D_h \tag{10}$$

where D_h is the hydraulic diameter of the cloud.

For a more accurate calculation of L, expression (9) has to be evaluated, usually by numerical methods. The only case in which it can be solved analytically is that of a spherical cloud, for which [3]:

$$KL = -\ln\left[\frac{8}{9(KD_h)^2}\{1-(1+\tfrac{3}{2}KD_h)\exp(-\tfrac{3}{2}KD_h)\}\right]. \tag{11}$$

However, a numerical integration of equation (9) shows that for spheres and infinite cylinders KL varies with KD_h in almost exactly the same way (Table 1). Hence equation (11) can be used for clouds of intermediate geometries and is to be preferred to equation (10) as the latter was derived

Table 1. Mean beam lengths of sphere and infinite cylinders

KD_h	KL (sphere)	KL (∞ cylinders)
0.2	0.198	0.195
1.0	0.93	0.91
3.0	2.4	2.3
5.0	3.4	3.4
10.0	4.7	4.8
100.0	9.3	9.4

primarily for volumes of absorbing gas, where multiple reflections are not present. Note that L tends to D_h as KL tends to 0, a fact which equation (10) fails to take into account.

RADIANT HEAT TRANSFER TO INDIVIDUAL PARTICLES

As a test of the validity of equation (8), we consider the case of a cloud of uniform-sized spherical particles at uniform temperature in black surroundings. The total heat-transfer rate Q from the surroundings to the cloud is given by

$$Q = \sigma\alpha_c A_c(T_r^4 - T_p^4) \tag{12}$$

while the mean heat-transfer rate to each of the $V_c n$ particles in the cloud is given by

$$q = \frac{Q}{V_c n} = \sigma\pi D_p^2 \alpha_p S(T_r^4 - T_p^4) \tag{13}$$

where S is a shape factor accounting for shielding effects (where particles hide each other from the radiation) and multiple reflection effects. A comparison of equations (12) and (13) gives

$$S = \frac{\alpha_c A_c}{V_c n\pi D_p^2 \alpha_p}$$

$$S = \frac{\alpha_c}{\alpha_p}\cdot\frac{1}{KD_h}. \tag{14}$$

For very dilute cloud, $KD_h \to 0$, $KL \to KD_h$ [from equation (11)], $\alpha_c \to \alpha_p KD_h$ [from equation (8)], and $S \to 1$ [equation (14)], as expected. Exactly the same result holds when $\alpha_p \to 0$, which means that if the particles are highly reflecting, the multiple reflections compensate for the shielding effect.

Table 2. Comparison of present and previous methods

α_p	KD_h	S (present method)	E (previous method)
0.1	1	0.91	0.86
	2	0.85	0.82
	5	0.72	0.72
	10	0.53	0.59
0.5	1	0.74	0.72
	2	0.58	0.59
	5	0.33	0.36
0.8	1	0.65	0.64
	2	0.46	0.48
	5	0.23	0.24

Table 2 gives some values of S as calculated by the present method [equations (8), (11) and (14)], and by previous methods [equations (2), (10) and (14)]. As expected, the two methods give divergent results for low particle absorptivities α_p. As the previous method ignores multiple reflections, it predicts low values for S in moderately dense clouds ($KD_h < 5$), but a high value for S in very dense clouds ($KD_h > 5$), when a large part of the incident radiation is "bounced off" the cloud at the boundary. It can also be seen that in the previous method S depends only on the product $\alpha_p KD_h$, while in the present method it is a function of both α_p and KD_h in different manners.

The present method is limited by the assumption that L' remains the same for successive reflections, and is equal to $L/2$. Strictly speaking, this holds only when reflection is isotropic and when the cloud is dilute. However, for moderately dense cloud it is expected that the present method would be more reliable than previous methods. For very dense clouds, the limiting absorptivity can be calculated from α_p in a different manner [1].

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